Esercizio: struttura due volte iperstatica con metodo delle forze (3 dicembre 2014)

Curvature anelastiche

$$\text{kbAB}[x\_] = 0$$

$$0$$

$$\text{kbBC}[x\_] = -\frac{2\,\alpha\,\Delta T}{h}$$

$$-\frac{2\,\alpha\,\Delta T}{h}$$

Momenti del problema zero

$$\text{M0AB}[x\_] = -F\,x$$

$$-F\,x$$

$$\text{M0BC}[x\_] = -F\,l - p\,\frac{x^2}{2}$$

$$-F\,l - \frac{p\,x^2}{2}$$

Momenti del problema uno

$$\text{M1AB}[x\_] = 0$$

$$0$$

$$\text{M1BC}[x\_] = x$$

$$x$$

Momenti del problema due

$$\text{M2AB}[x\_] = -1$$

$$-1$$

$$\text{M2BC}[x\_] = -1$$

$$-1$$

Coefficienti di influenza

$$\eta 11 = \int_0^l \frac{\text{M1AB}[x]^2}{EI}\,dx + \int_0^l \frac{\text{M1BC}[x]^2}{EI}\,dx$$

$$\frac{l^3}{3\,EI}$$

$$\eta 22 = \int_0^l \frac{\text{M2AB}[x]^2}{EI}\,dx + \int_0^l \frac{\text{M2BC}[x]^2}{EI}\,dx$$

$$\frac{2\,l}{EI}$$

$$\eta 12 = \int_0^l \frac{\mathbf{M1AB[x]\ M2AB[x]}}{\mathbf{EI}} \, dx + \int_0^l \frac{\mathbf{M1BC[x]\ M2BC[x]}}{\mathbf{EI}} \, dx$$

$$-\frac{l^2}{2\ EI}$$

$$\eta 10 = \int_0^l \frac{\mathbf{M1AB[x]\ M0AB[x]}}{\mathbf{EI}} \, dx + \int_0^l \frac{\mathbf{M1BC[x]\ M0BC[x]}}{\mathbf{EI}} \, dx$$

$$-\frac{F\ l^3}{2\ EI} - \frac{l^4\ p}{8\ EI}$$

$$\eta 20 = \int_0^l \frac{\mathbf{M2AB[x]\ M0AB[x]}}{\mathbf{EI}} \, dx + \int_0^l \frac{\mathbf{M2BC[x]\ M0BC[x]}}{\mathbf{EI}} \, dx$$

$$\frac{3\ F\ l^2}{2\ EI} + \frac{l^3\ p}{6\ EI}$$

$$\eta 1b = \int_0^l \mathbf{M1AB[x]\ kbAB[x]} \, dx + \int_0^l \mathbf{M1BC[x]\ kbBC[x]} \, dx$$

$$-\frac{l^2\ \alpha\ \Delta T}{h}$$

$$\eta 2b = \int_0^l \mathbf{M2AB[x]\ kbAB[x]} \, dx + \int_0^l \mathbf{M2BC[x]\ kbBC[x]} \, dx$$

$$\frac{2\ l\ \alpha\ \Delta T}{h}$$

Risoluzione equazioni di compatibilità cinematica

soluzione = Solve[
{η11 χ1 + η12 χ2 + η10 + η1b == 0, η12 χ1 + η22 χ2 + η20 + η2b == 0}, {χ1, χ2}] // Simplify

$$\left\{\left\{\chi 1 \to \frac{1}{5}\left(3\ F + 2\ l\ p + \frac{12\ EI\ \alpha\ \Delta T}{h\ l}\right),\ \chi 2 \to \frac{1}{60}\left(-36\ F\ l + l^2\ p - \frac{24\ EI\ \alpha\ \Delta T}{h}\right)\right\}\right\}$$

Sostituzione valori numerici

soluzione /. {EI → 64 000, p → 20, F → 40, ΔT → 20, α → 10^{-5}, h → 0.4, l → 4}

$\{\{\chi 1 \to 75.2,\ \chi 2 \to -103.467\}\}$